COMPUTING TOPOLOGICAL INDICES OF DIRECT INTERCONNECTION NETWORKS

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ABSTRACT. *A direct interconnection network (IN) of a multiprocessor system is represented by a connected graph whose vertices represent processing nodes and edges represent communication links. A processing node (PN) usually consists of one or more processors, local memory, and communication router.*
The butterfly and Benes networks are important class of multistage direct interconnection networks which are defined based on the schemes that connect the units of a multiprocessing system and needs n stages to connect 2^n processors; at each stage a switch is thrown, depending on a particular bit in the addresses of the processors being connected. In this paper, degree based topological indices of these direct interconnection networks are strong-minded.

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1 INTRODUCTION AND PRELIMINARY RESULTS

A single number which characterizes the graph of a molecular structure is called a graph-theoretical invariant or topological index. Topological indices have been found as a base to understand the topologies of interconnection networks. In addition, these numerical parameters discuss the structural properties of these important architectures.

A *direct interconnection network* (IN) of a multiprocessor system is represented by a connected graph whose vertices represent processing nodes and edges represent communication links. A processing node (PN) usually consists of one or more processors, local memory, and communication router. An IN should transfer a maximum number of messages in the shortest time with minimum cost and maximal reliability. Clearly, any design of an IN is a tradeoff of various contradictory requirements.

Butterfly graphs are defined as the underlying graphs of Fast Fourier Transforms (FFT) networks which can perform the FFT very efficiently. The butterfly network consists of a series of switch stages and interconnection patterns, which allows 'n' inputs to be connected to 'n' outputs. The *Benes network* consists of back-to-back butterflies. As butterfly is known for FFT, Benes is known for permutation routing [2]. The butterfly and Benes networks are important multistage interconnection networks, which possess attractive topologies for communication networks [17]. They have been used in parallel computing systems such as IBM, SP1/SP2, MIT Transit Project, NEC Cenju-3 and used as well in the internal structures of optical couplers [16, 19]. The multistage networks have long been used as communication networks for parallel computing [15].

A graph $G(V,E)$ with vertex set V and edge set E is connected, if there exist a connection between any pair of vertices in G. The *degree* of a vertex in a network graph is the number of vertices which are connected to that fixed vertex by the edges.

In this article, G is considered to be a connected graph with vertex set $V(G)$ and edge set $E(G)$, d_u is the degree of vertex $u \in V(G)$. The notations used in this article are mainly taken from books [4, 10].

The general Randić index was proposed by Bollobás and Erdös [3] and Amic et al. [1] independently, in 1998. Then it has been extensively studied by both mathematicians and theoretical chemists [13].

Definition 1.1. *For a graph G, the general Randić index $R_\alpha(G)$ is the sum of $(d_u d_v)^\alpha$ over all edges $e = uv \in E(G)$ defined as*

$$R_\alpha(G) = \sum_{uv\in E(G)} (d_u d_v)^\alpha$$

where $\alpha(\alpha \neq 0)$ is an arbitrary real number.

With motivation from the Randić index, a closely related variant of the Randić connectivity index called the sum-connectivity index was recently proposed by Zhou and Trinajstić [20] in 2009.

Definition 1.2. *The sum- connectivity index $\chi(G)$ of graph G is defined as follows:*

$$X(G) = \sum_{uv\in E(G)} (d_u + d_v)^{-\frac{1}{2}}$$

and the general sum-connectivity index $X_\alpha(G)$ was defined as follows:

$$X_\alpha(G) = \sum_{uv\in E(G)} (d_u + d_v)^{-\alpha}$$

where $\alpha(\alpha \neq 0)$ *is an arbitrary real number.*

$X(G)$ is a graph-based molecular structure descriptor. The sum-connectivity index has been found to correlate well with π-electronic energy of benzenoid hydrocarbons. The the sum-connectivity index and original Randić connectivity index are highly intercorrelated molecular descriptors. Some mathematical properties of the sum-connectivity and general sum-connectivity are given in [5, 6].

An important topological index introduced about forty years ago by *Ivan Gutman* and *Trinajstić* is the *Zagreb index* or more precisely first zagreb index denoted by $M_1(G)$ and was defined as the sum of degrees of end vertices of all edges of G.

Definition 1.3. *The first zagreb index is defined as*

$$M_1(G) = \sum_{uv \in E(G)} (d_u + d_v)$$

One of the well-known connectivity topological index is *atom-bond connectivity* (ABC) index introduced by *Estrada* et al. in [7].

Definition 1.4. *For a graph* G*, the* ABC *index is defined as*

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$$

Another well-known connectivity topological descriptor is *geometric-arithmetic* (GA) index which was introduced by *Vukičević* et al. in [18].

Definition 1.5. *Consider a graph* G*, then its* GA *index is defined as*

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{(d_u + d_v)}$$

The fourth version of ABC index is introduced by *Ghorbani* et al. [8] recently in 2010.

Definition 1.6. *Let* G *be a graph, then its fourth* ABC *index is defined as*

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}$$

Recently fifth version of GA index is proposed by *Graovac* et al. [9] in 2011.

Definition 1.7. *For a graph* G*, the fifth version of* GA *index is defined as*

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{(S_u + S_v)}$$

Following lemma is important throughout our discussion.

Lemma 1.1. [12] *For any graph* G*, when* $\alpha = 1$*,* $H(G) = 2X_1(G)$.

2 MAIN RESULTS

In this article, we study the general sum-connectivity, Harmonic, First Zagreb, ABC_4 and GA_5 indices and give closed formulas of these indices for two important classes of interconnection networks named as butterfly and Benes networks. For further study of topological indices of networks see [11, 14].

2.1 Results for butterfly network

The most popular bounded-degree derivative network of the hypercube is the butterfly network. The set V of vertices of an r-dimensional butterfly network correspond to pairs $[w, i]$, where i is the dimension or level of a node $(0 \leq i \leq r)$ and w is an r-bit binary number that denotes the row of the node. Two nodes $[w, i]$ and $[w,' i']$ are linked by an edge if and only if $i' = i + 1$ and either:

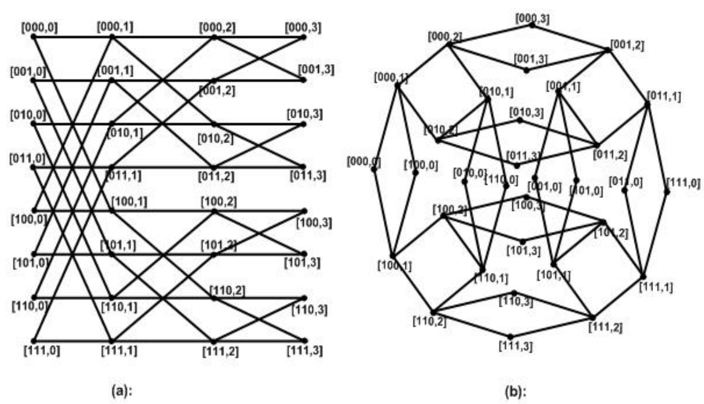
1. w and w are identical, or
2. w and w differ in precisely the i th bit.

The edges in the network are undirected. An r-dimensional butterfly network is denoted by $BF(r)$. Manuel et el. [17] proposed the diamond representations of these networks. The normal and diamond representations of 3-dimensional butterfly network are given in Fig. 1. The vertex and edge cardinalities are $2^r(r+1)$ and $r2^{r+1}$ respectively.

Figure 1: (a)Normal representation of butterfly $BF(3)$ **(b)Diamond representation of butterfly** $BF(3)$

Lemma 2.1.1. *For any graph* G*, when* $\alpha = -1$*,* $M_1(G) = \chi_{-1}(G)$.

Now we compute certain degree based topological indices for



butterfly network. Following theorem presents the analytically closed formula of general sum-connectivity index $X_\alpha(G)$ with $\alpha = 1, -1, \frac{1}{2}, -\frac{1}{2}$ for this network.

Theorem 2.1.1. *Consider the butterfly network* $BF(r)$*, then its general sum-connectivity index is equal to*

$X_\alpha(BF(r)) =$

$$\begin{cases} (\frac{r}{8}+\frac{1}{12})2^{r+1}, & \alpha = 1; \\ (\frac{r\sqrt{2}}{4}+\frac{\sqrt{6}}{3}-\frac{\sqrt{2}}{2})2^{r+1}, & \alpha = \frac{1}{2}; \\ (8r+2)2^{r+1}, & \alpha = -1; \\ (4r+2\sqrt{6}-8)2^{r+1}, & \alpha = -\frac{1}{2}. \end{cases}$$

Proof. Consider H be the butterfly network with defining parameter r. The number of vertices and edges in H are $2^r(r+1)$ and $r2^{r+1}$ respectively. There are two types of edges in H based on degrees of end vertices of each edge. Table 1 shows such an edge partition of H.

Table 1: Edge partition of butterfly network $BF(r)$ based on degrees of end vertices of each edge.

(d_u, d_v) where $uv \in E(G)$	(2,4)	(4,4)
Number of edges	2^{r+2}	$2^{r+1}(r-2)$

For $\alpha = 1$

Now we apply the formula of $R_\alpha(G)$ for $\alpha = 1$.

$$X_1(G) = \sum_{uv \in E(G)} (d_u + d_v)^{-1}$$

By using edge partition given in table 1, we get,

$$X_1(H) = 2^{r+2}(\frac{1}{2+4}) + (2^{r+1}(r-2))(\frac{1}{4+4})$$

After simplifying, we get a non-linear expression in parameter r,

$$X_1(H) = (\frac{r}{8}+\frac{1}{12})2^{r+1}.$$

For $\alpha = \frac{1}{2}$

We apply the formula of $X_\alpha(G)$ for $\alpha = \frac{1}{2}$.

$$X_{\frac{1}{2}}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{(d_u + d_v)}}$$

By using edge partition given in table 1, we get this non-linear expression in parameter r,

$$X_{\frac{1}{2}}(H) = 2^{r+2}(\frac{1}{\sqrt{2+4}}) + (2^{r+1}(r-2))(\frac{1}{\sqrt{4+4}})$$

$$X_{\frac{1}{2}}(H) = (\frac{r\sqrt{2}}{4}+\frac{\sqrt{6}}{3}-\frac{\sqrt{2}}{2})2^{r+1}.$$

For $\alpha = -1$

We apply the formula of $X_\alpha(G)$ for $\alpha = -1$.

$$X_{-1}(G) = \sum_{uv \in E(G)} (d_u + d_v)$$

$$X_{-1}(H) = 2^{r+2}(2+4) + (2^{r+1}(r-2))(4+4)$$

$$X_{-1}(H) = (8r+2)2^{r+1}.$$

For $\alpha = -\frac{1}{2}$

We apply the formula of $X_\alpha(G)$ for $\alpha = -\frac{1}{2}$.

$$X_{-\frac{1}{2}}(G) = \sum_{uv \in E(G)} \sqrt{(d_u + d_v)}$$

$$X_{-\frac{1}{2}}(H) = 2^{r+2}(\sqrt{2+4}) + (2^{r+1}(r-2))(\sqrt{4+4})$$

$$X_{-\frac{1}{2}}(H) = (4r+2\sqrt{6}-8)2^{r+1}.$$

The subsequent corollaries of above theorem and lemmas 1.1 and 2.1.1 are following.

Corollary 2.1.1. *For butterfly network $BF(r)$, the Harmonic index is equal to*

$$H(BF(r)) = (\frac{r}{2}+\frac{1}{6})2^{r+1}.$$

Corollary 2.1.1. *For butterfly network $BF(r)$, the first zagreb index is equal to*

$$M_1(BF(r)) = (8r+2)2^{r+1}.$$

The ABC and GA indices of this network have already been studied in other papers. We compute ABC_4 and GA_5 indices of this r-dimensional network. We need an edge partition of this graph based on degree sum of neighbors of end vertices of each edge. Table 2 shows such a partition of this graph.

Table 2: Edge partition of graph of r-dimensional

(S_u, S_v) where $uv \in E(G)$	Number of edges
(8,12)	2^{r+2}
(12,16)	2^{r+2}
(16,16)	$(r-4)2^{r+1}$

butterfly network based on degree sum of vertices lying at unit distance from end vertices of each edge.

Following theorem exhibits the analytically closed result of ABC_4 index for this network.

Theorem 2.1.2. *Consider the r-dimensional butterfly network, then its ABC_4 index is a non-linear expression in parameter r,*

$$ABC_4(BF(r)) = (r\frac{\sqrt{30}}{16}-\frac{\sqrt{30}}{4}+\frac{\sqrt{3}}{2}+\frac{\sqrt{78}}{12})2^{r+1}.$$

Proof. Consider H be the r-dimensional butterfly network with defining parameter r. The number of vertices and edges in H are $2^r(r+1)$ and $r2^{r+1}$ respectively. There are three types of edges in H based on degrees of end vertices of each edge. Table 2 shows such an edge partition of H.

We know

$$ABC_4(G) = \sum_{uv\in E(G)} \sqrt{\frac{S_u+S_v-2}{S_uS_v}}$$

$$ABC_4(H) = 2^{r+2}\sqrt{\frac{8+12-2}{8\times 12}} + 2^{r+2}\sqrt{\frac{12+16-2}{12\times 16}} +$$

$$((r-4)2^{r+1})\sqrt{\frac{16+16-2}{16\times 16}}$$

By doing some calculation, we get this non-linear expression in defining parameter,

$$ABC_4(BF(r)) = (r\frac{\sqrt{30}}{16} - \frac{\sqrt{30}}{4} + \frac{\sqrt{3}}{2} + \frac{\sqrt{78}}{12})2^{r+1}.$$

In the following theorem, we compute GA_5 index of r-dimensional butterfly network.

Theorem 2.1.3. *Consider the r-dimensional butterfly network, the its GA_5 index is a non-linear expression in parameter r,*

$$GA_5(BF(r)) = (\frac{4\sqrt{6}}{5} + \frac{8\sqrt{3}}{7} + r - 4)2^{r+1}.$$

Proof. Let H be the r-dimensional butterfly network. We easily prove it by using edge partition given in table 2. We know

$$GA_5(G) = \sum_{uv\in E(G)} \frac{2\sqrt{S_uS_v}}{(S_u+S_v)}$$

$$GA_5(H) = 2^{r+2}(\frac{2\sqrt{8\times 12}}{8+12}) + 2^{r+2}(\frac{2\sqrt{12\times 16}}{12+16}) +$$

$$((r-4)2^{r+1})(\frac{2\sqrt{16\times 16}}{16+16})$$

By doing some calculation, we get this non-linear expression in parameter r

$$GA_5(BF(r)) = (\frac{4\sqrt{6}}{5} + \frac{8\sqrt{3}}{7} + r - 4)2^{r+1}.$$

Now we deal with another type of important direct IN named as Benes network. We give analytically closed results for topological indices of these networks which provide us a base to understand their topologies of these well-designed architectures.

2.2 Results for Benes network

An r-dimensional Benes network is nothing but back-to-back butterflies. An r-dimensional Benes network has $2r+1$ levels, each level with $2r$ nodes. The level 0 to level r nodes in the network form an r-dimensional butterfly. The middle level of the Benes network is shared by these butterflies. An r-dimensional Benes is denoted by $B(r)$. Manuel et al. proposed the diamond representation of the Benes network also [17]. Fig. 2 shows the normal representation of $B(3)$ network, while diamond representation of $B(3)$ is depicted in Fig. 3. The number of vertices and number of edges in an r-dimensional Benes network are $2^r(2r+1)$ and $r2^{r+2}$ respectively.

Figure 2: Normal representation of Benes network $B(3)$.

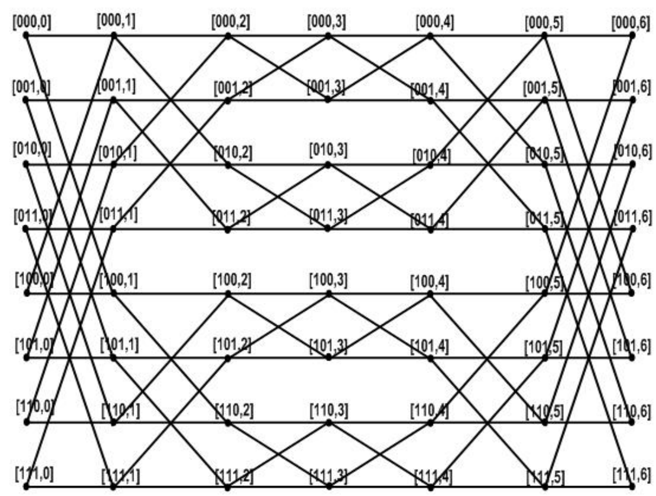
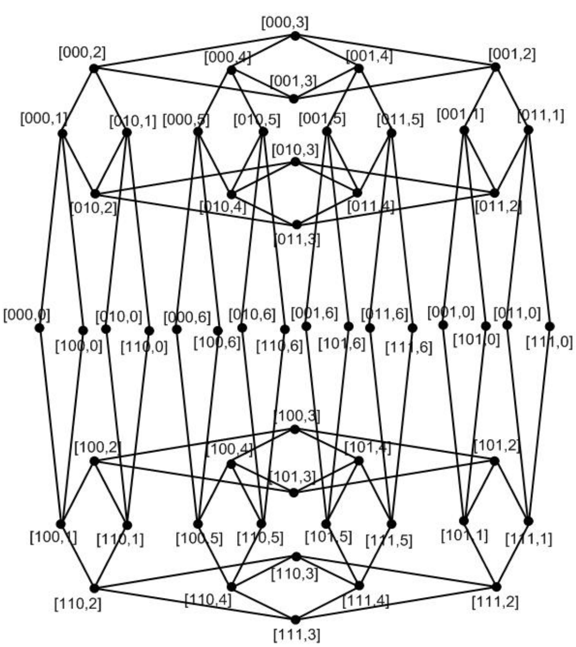


Figure 3: Diamond representation of Benes network



$B(3)$.

Now we compute certain degree based topological indices for Benes network. Following theorem presents the analytically closed formula of general sum-connectivity index $X_\alpha(G)$ with $\alpha = 1, -1, \frac{1}{2}, -\frac{1}{2}$ for this network.

Theorem 2.2.1. *Consider the Benes network $B(r)$, then its general sum-connectivity index is equal to*

$$X_\alpha(B(r)) = \begin{cases} (\frac{r}{8}+\frac{1}{24})2^{r+2}, & \alpha = 1; \\ (\frac{r\sqrt{2}}{4}+\frac{\sqrt{6}}{6}-\frac{\sqrt{2}}{4})2^{r+2}, & \alpha = \frac{1}{2}; \\ (8r-2)2^{r+2}, & \alpha = -1; \\ (4r+\sqrt{6}-4)2^{r+2}, & \alpha = -\frac{1}{2}. \end{cases}$$

Proof. Consider H be the Benes network with defining parameter r. The number of vertices and edges in H are $2^r(2r+1)$ and $r2^{r+2}$ respectively. There are two types of edges in H based on degrees of end vertices of each edge. Table 3 shows such an edge partition of H.

Table 3: Edge partition of Benes network $B(r)$ based on degrees of end vertices of each edge.

(d_u, d_v) where $uv \in E(G)$	(2,4)	(4,4)
Number of edges	2^{r+2}	$2^{r+2}(r-1)$

For $\alpha = 1$

Now we apply the formula of $R_\alpha(G)$ for $\alpha = 1$.

$$X_1(G) = \sum_{uv\in E(G)} (d_u + d_v)^{-1}$$

By using edge partition given in table 3, we get,

$$X_1(H) = 2^{r+2}(\frac{1}{2+4}) + (2^{r+2}(r-1))(\frac{1}{4+4})$$

After simplifying, we get a non-linear expression in parameter r,

$$X_1(H) = (\frac{r}{8}+\frac{1}{12})2^{r+1}.$$

For $\alpha = \frac{1}{2}$

We apply the formula of $X_\alpha(G)$ for $\alpha = \frac{1}{2}$.

$$X_{\frac{1}{2}}(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{(d_u + d_v)}}$$

By using edge partition given in table 3, we get this non-linear expression in parameter r,

$$X_{\frac{1}{2}}(H) = 2^{r+2}(\frac{1}{\sqrt{2+4}}) + (2^{r+2}(r-1))(\frac{1}{\sqrt{4+4}})$$

$$X_{\frac{1}{2}}(H) = (\frac{r\sqrt{2}}{4}+\frac{\sqrt{6}}{6}-\frac{\sqrt{2}}{4})2^{r+2}.$$

For $\alpha = -1$

We apply the formula of $X_\alpha(G)$ for $\alpha = -1$.

$$X_{-1}(G) = \sum_{uv\in E(G)} (d_u + d_v)$$

$$X_{-1}(H) = 2^{r+2}(2+4) + (2^{r+2}(r-1))(4+4)$$

$$X_{-1}(H) = (8r-2)2^{r+2}.$$

For $\alpha = -\frac{1}{2}$

We apply the formula of $X_\alpha(G)$ for $\alpha = -\frac{1}{2}$.

$$X_{-\frac{1}{2}}(G) = \sum_{uv\in E(G)} \sqrt{(d_u + d_v)}$$

$$X_{-\frac{1}{2}}(H) = 2^{r+2}(\sqrt{2+4}) + (2^{r+2}(r-1))(\sqrt{4+4})$$

$$X_{-\frac{1}{2}}(H) = (4r+\sqrt{6}-4)2^{r+2}.$$

The subsequent corollaries of above theorem and lemmas 1.1 and 2.1.1 are following.

Corollary 2.1.1. *For Benes network $B(r)$, the Harmonic index is equal to*

$$H(B(r)) = (\frac{r}{4}+\frac{1}{12})2^{r+2}.$$

Corollary 2.1.1. For Benes network $B(r)$, the first zagreb index is equal to

$$M_1(B(r)) = (8r-2)2^{r+2}.$$

The ABC and GA indices of this network have already been calculated in other papers. We compute ABC_4 and GA_5 indices of this r-dimensional network. We need an edge partition of this graph based on degree sum of neighbors of end vertices of each edge. Table 4 shows such a partition of this graph.

Table 4: Edge partition of graph of r-dimensional Benes network based on degree sum of vertices lying at unit distance from end vertices of each edge.

(S_u, S_v) where $uv \in E(G)$	Number of edges

(8,12)	2^{r+2}
(12,16)	2^{r+2}
(16,16)	$(r-2)2^{r+2}$

Following theorem exhibits the analytically closed result of ABC_4 index for this network.

Theorem 2.2.2. *Consider the r-dimensional Benes network, then its ABC_4 index is a non-linear expression in parameter r,*

$$ABC_4(B(r)) = (r\frac{\sqrt{30}}{16} - \frac{\sqrt{30}}{16} + \frac{\sqrt{3}}{4} + \frac{\sqrt{78}}{24})2^{r+2}.$$

Proof. Consider H be the r-dimensional Benes network with defining parameter r. The number of vertices and edges in H are $2^r(2r+1)$ and $r2^{r+2}$ respectively. There are three types of edges in H based on degrees of end vertices of each edge. Table 4 shows such an edge partition of H.

We know

$$ABC_4(G) = \sum_{uv\in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}$$

$$ABC_4(H) = 2^{r+2}\sqrt{\frac{8+12-2}{8\times 12}} + 2^{r+2}\sqrt{\frac{12+16-2}{12\times 16}} +$$

$$((r-2)2^{r+2})\sqrt{\frac{16+16-2}{16\times 16}}$$

By doing some calculation, we get this non-linear expression in defining parameter,

$$ABC_4(B(r)) = (r\frac{\sqrt{30}}{16} - \frac{\sqrt{30}}{16} + \frac{\sqrt{3}}{4} + \frac{\sqrt{78}}{24})2^{r+2}.$$

In the following theorem, we compute GA_5 index of r-dimensional Benes network.

Theorem 2.2.3. *Consider the r-dimensional Benes network, the its GA_5 index is equals to*

$$GA_5(B(r)) = (\frac{2\sqrt{6}}{5} + \frac{4\sqrt{3}}{7} + r - 1)2^{r+2}.$$

Proof. Let H be the r-dimensional Benes network. We easily prove it by using edge partition given in table 4. We know

$$GA_5(G) = \sum_{uv\in E(G)} \frac{2\sqrt{S_u S_v}}{(S_u + S_v)}$$

$$GA_5(H) = 2^{r+2}(\frac{2\sqrt{8\times 12}}{8+12}) + 2^{r+2}(\frac{2\sqrt{12\times 16}}{12+16}) +$$

$$((r-2)2^{r+2})(\frac{2\sqrt{16\times 16}}{16+16})$$

By doing some calculation, we get this non-linear expression in parameter r

$$GA_5(B(r)) = (\frac{2\sqrt{6}}{5} + \frac{4\sqrt{3}}{7} + r - 1)2^{r+2}.$$

3 CONCLUDING REMARKS

In this paper, certain degree based topological indices, namely general sum-connectivity index, Harmonic index, first Zagreb index, fourth atomic-bond connectivity index (ABC_4) and fifth geometric-arithmetic index (GA_5) for two important class of networks were studied for the first time. We computed analytically closed results of these degree based topological indices for butterfly and Benes interconnection networks. These results provide a base to understand the topology of these networks. In future, we are interested to study fat tree network and extended fat tree network.

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